SMITH

Laboratory Methods for Measuring Electrical Capacity

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LABORATORY METHODS FOR MEASURING ELECTRICAL CAPACITY

BY

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THESIS

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

ORRIN HAROLD SMITH

ENTITLED LABORATORY METHODS FOR MEASURING ELECTRICAL CAPACITY

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Master of Arts in Physics

Chas. T. Knipp In Charge of Major Work

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Recommendation concurred in:

Committee

on

Final Examination

C O N T E N T S

LABORATORY METHODS FOR MEASURING ELECTRICAL CAPACITY

INTRODUCTION

The simple properties of a condenser and the methods of measuring capacity, have long been known having been so widely and so thoroughly discussed by early investigators that it has seemed almost absurd to modern physicists to call them or their application in question. Yet, we find one of these simple properties affecting our methods of measurement of capacity very materially and physicists have been very slow in making allowance for it. The phenomenon referred to is that of a residual discharge.
This phenomenon has been known in connection with the Leyden jar for a considerable time. Faraday, as early as 1831-38, carried on extended experiments on the properties of dielectrics of condensers; Cavendish, even as far back as 1771, determined the value of K for several substances, this value of K being named by Faraday in 1831 the "specific inductive capacity" of a substance. Writers since them have frequently mentioned this particular property but only a few have suggested that a correction should be made for it in their methods of measurement. Experiments are performed in the junior laboratory at the University of

Illinois which show a recognition of the existence of residual discharge but the absolute methods of measurement have not been corrected with reference to it. In the comparison of an unknown capacity with a standard nothing is said about the effect of a high absorbing power in the dielectric of the unknown capacity, although condensers having a dielectric of high absorbing power are avoided in connection with these experiments.

Professor Anthony Zeleny of the University of Minnesota was the first to define the different kinds of capacities of condensers* in the light of the effect of an absorbing power in the dielectric on the ballistic throw of a galvanometer through which it is discharged. He divides the kinds of capacities under four heads:- "True" "Free Charge", "Ordinary" and "Effective" capacities. The basis for this classification will be shown subsequently.

The purpose of this thesis is to investigate laboratory methods of measuring capacities in the light of Professor Zeleny's classification, in order to modify them so that consistent results may be obtained by students in the laboratory. It is desired, also, to investigate the bearing of the different factors in the methods under consideration on the results obtained and to define them if possible as to conditions for greatest accuracy. Some attention also, will be given to phenomena which occur in

*Phys. Rev., Vol.XXVII, p 65, 1908.

Standard Mica Condenser---.4 M.F. Parafin Condenser ---.?

	Arc	Time-Sec.	d		Arc	TIME	Deflections
Set A.	394	.0	0	Set B.	394	.0	0
	394.5	.00039	47		394.5	.00039	43
	395	.00077	49		395	.00077	55
	396	.00154	49		396	.00154	56.5
	397	.00231	49.5		397	.00231	56.5
	400	.00462	50		400	.00462	57
	410	.01231	50		410	.01231	57
	430	.02909	50		430	.02909	57
	1/4 T	1.465	50		1/4 T	1.465	58.5

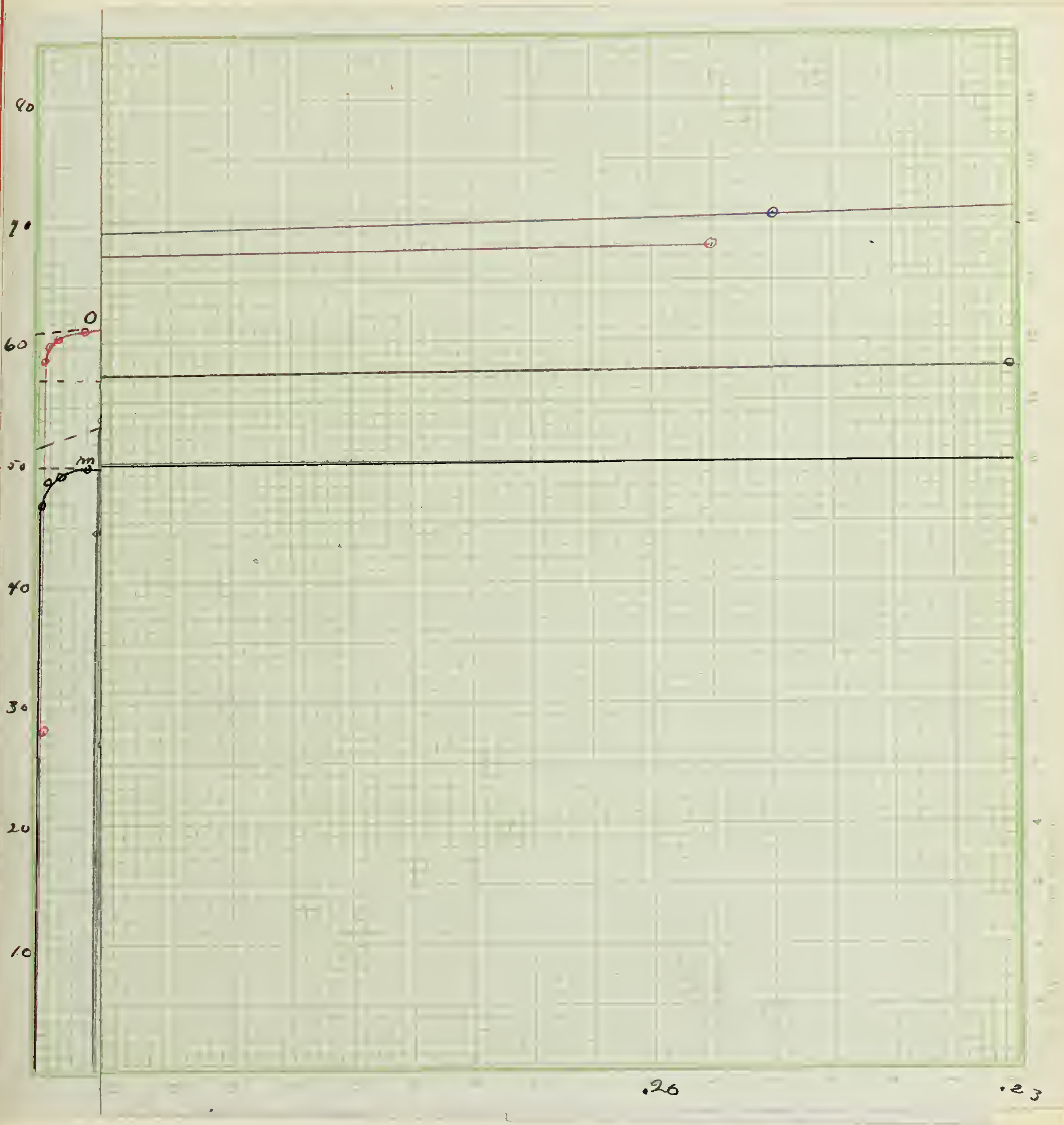
Paper Condenser---.5 M.F. Old Condenser-----.? M.F.

Set C.-	394	.0	0	Set D.--	394	.0	0
	394.5	.00039	28		394.5	.00039	27
	395	.00077	60		395	.00077	40
	396	.00154	60.5		396	.00154	47.5
	397	.00231	61		397	.00231	48.5
	400	.00462	61		400	.00462	50.5
	410	.01231	62		410	.01231	57
	430	.02909	63		430	.02909	60
	470	.06273	64		470	.06273	64
	530	.11571	65		530	.11571	67
	600	.18200	67		600	.18200	69
	675	.26668	68		675	.26668	70.5
	1/4 T	1.465	77		1/4 T	1.465	74

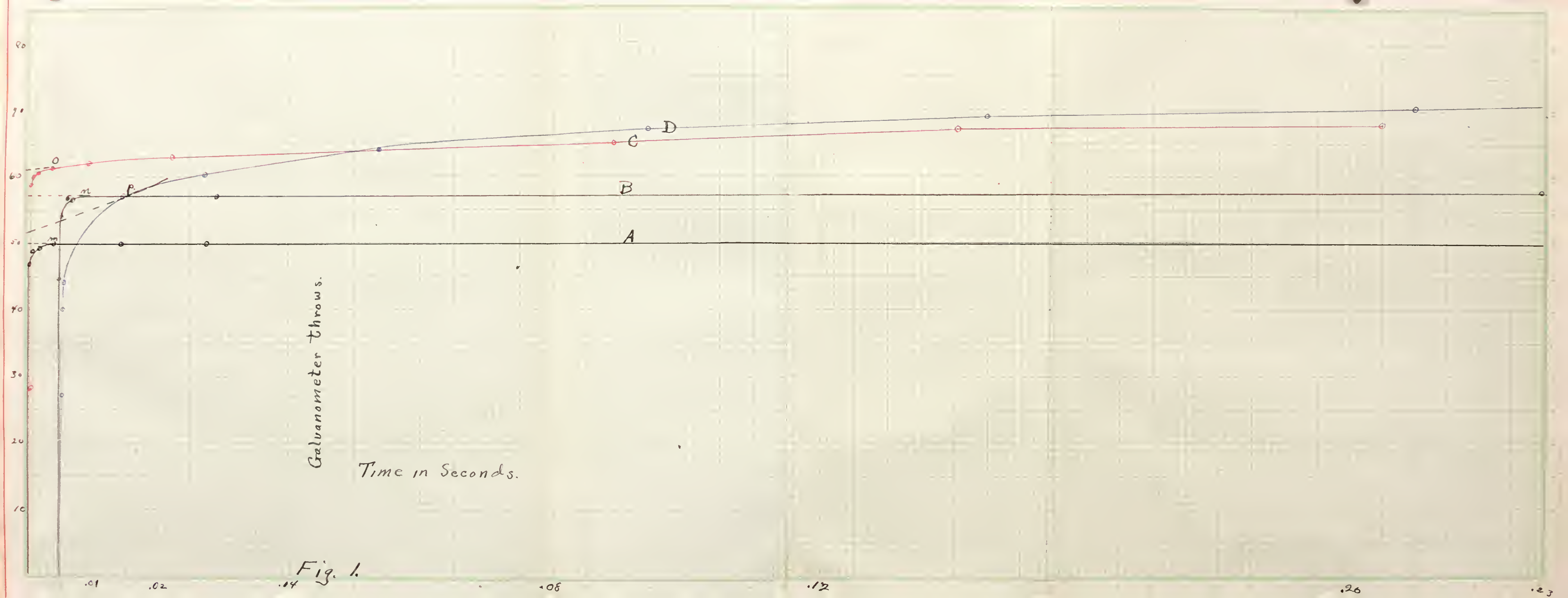
Type H. Leeds and Northrup Galvanometer-------Pl.2717 B.

$T=5.86$ $R_g=499$ $\lambda=.0196$ E.M.F. = One dry cell

TABLE I



80
70
60
50
40
30
20
10
O
m
.20
.23



80
70
60
50
40
30
20
10
D
C
B
A
o
n
p
m
Galvanometer throws.
Time in Seconds.
.01
.02
.04
.08
.12
.20
.23

Fig. 1.

connection with the factors. The investigation will be confined to absolute methods only.

Figure 1. shows curves plotted with time in seconds as abscissae and galvanometer throws in millimeters as ordinates. Table I, contains the data corresponding to these curves. Curve A, was obtained from a standard mica condenser, Curve B. from a parafin, Curve C. from a new Leeds and Northrop paper-parafin, having a nominal capacity of 1/2 microfarad, and Curve D. from an old paper-parafin condenser. The standard mica condenser could be varied from .05 to 1. microfarad while the condensers for which B and D are the curves had no definite nominal value.

From the curves it is evident that the free charge is liberated in an exceedingly short interval of time, nearly all being liberated, in the case of all four condensers, in .00077 second. Curve A, from the standard mica condenser, after the point m. runs horizontal even up to 1.46 seconds, the time of a quarter period of the galvanometer coil. It will be seen later in connection with the data taken when using a galvanometer of a longer period, that there is a slight rise in the curve between the point m and the farther end. In Curve B, we can see, even when using this galvanometer, a little difference between the throws for 0.0015 second and 1.46 seconds. We find the rise, in Curve C. greater and in Curve D. greater still. On all of these curves it is not hard to determine pretty closely

by inspection the point where the free charge is entirely liberated and from which point the subsequent rise of the curve is due to the liberation of the absorbed charge. For example, points m, n, o, and p, corresponding to curves, A, B, C and D respectively, mark the points where the full measure of the free charge is out. Now, if we assume, as Professor Zeleny does, that the absorbed charge has been liberated at the same rate since the free charge began to go off, as just succeeding the points m, n, o and p on the curves, and if we project straight lines backward from the curves which are tangent to the curves at these points, the ordinates of the points where these tangent lines cut the axis of ordinates may then be regarded as representing the "true" capacity of the condenser; the ordinates of the points m, n, o and p as representing the "free charge" capacity; the "ordinary" capacity that represented by the ordinate of the point at the end of the quarter period of the galvanometer; and the "effective" that of the ordinate of a point corresponding to any time desired to suit the conditions of a particular experiment or purpose.

We see from the curves (Figure 1) that what happens when we close a circuit on a charged condenser is a sudden pulse due to the free charge and then a gradual liberation of the absorbed charge which is equivalent to a current of electricity and which affects a galvanometer as a current

does. When using a ballistic galvanometer for the measurement of a quantity of electricity, the formula used holds only when all the electricity has passed through the coil before it has moved at all,* or at least through an appreciable angle.≠ This statement is evidenced by the fact that when an Ayton-Mather galvanometer, which has a very light, quickly moving system, was used ballistically for the measurement of capacity, the values were off by a large percent.⊕ It is evident then that the ordinary value of a capacity is not a true capacity at all, only manifesting itself as a capacity in its effects on the galvanometer but from the above statement we find that what affects a galvanometer as if it were a discharge is not a discharge but an equivalent of a current of gradually diminishing strength. That is, after the free charge has given the coil an impulse, the absorbed charge then adds a continued force which urges the coil beyond the point where the impulse

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*Electrical Measurements - Carhart and Patterson p.207

≠Practical Electricity and Magnetism - Henderson p.211

Hand book of Electrical Testing - Kempe p.68

⊕Another objection to the Ayrton - Mather galvanometer for laboratory methods is that no pier could be found in the laboratory that was steady enough for its use.

would have taken it, and according to the theory, above stated in regard to the measurement of quantity with a ballistic galvanometer, this resultant throw is not the same as would be due to an equal quantity of electricity dicharged all at once. The absorbed charge liberated during the passage of the free charge may be considered a real discharge since it comes off with the free charge, having the same effect as the free charge and only acting to strengthen it.

It is interesting to note in connection with Curve D. which has a distinctively slow bend, being obtained from the old condenser, Professor F. E. Nipher's statement in regard to the specific inductive capacity of a dielectric. He says* "The specific inductive capacity of a substance is a very complex quantity. It is a function of the duration of charging and also of the time and duration of former charges. It is probable that the entire prior history of the dielectric is involved in the value of K for a given specimen at a given time". It will be seen further, that this distinctive slow bend is characteristic of a curve obtained by Professor Zeleny from a condenser which, he says, had been used for fiteen years.#

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* Electricity and Magnetism - Nipher p. 100.

#Phys. Rev. Vol. XXVII, p. 70, Fig. 6.

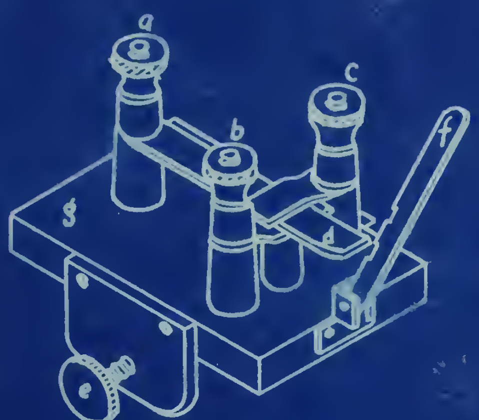


FIG 3

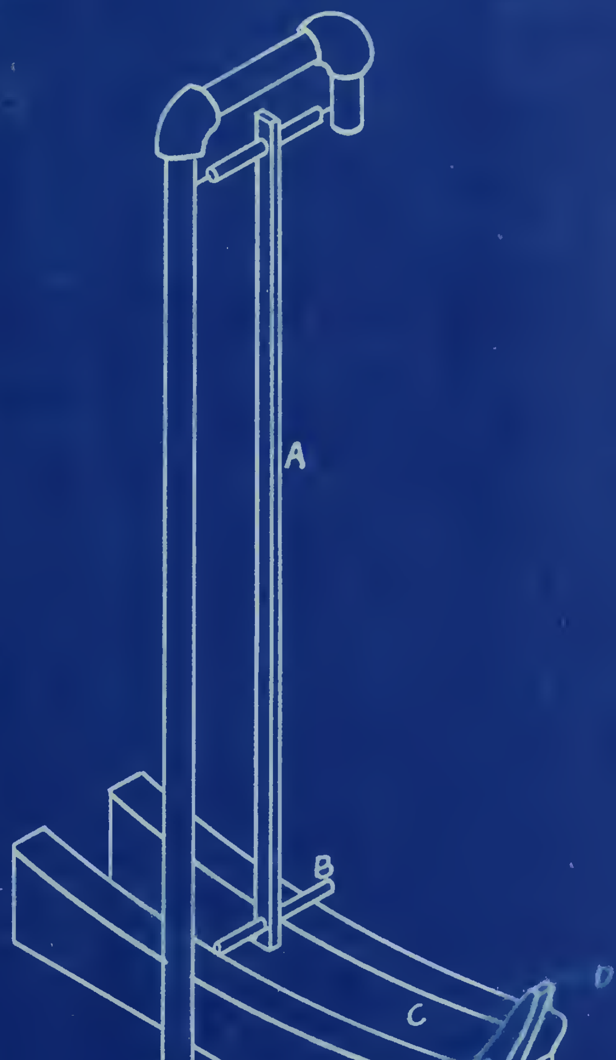


FIG 2

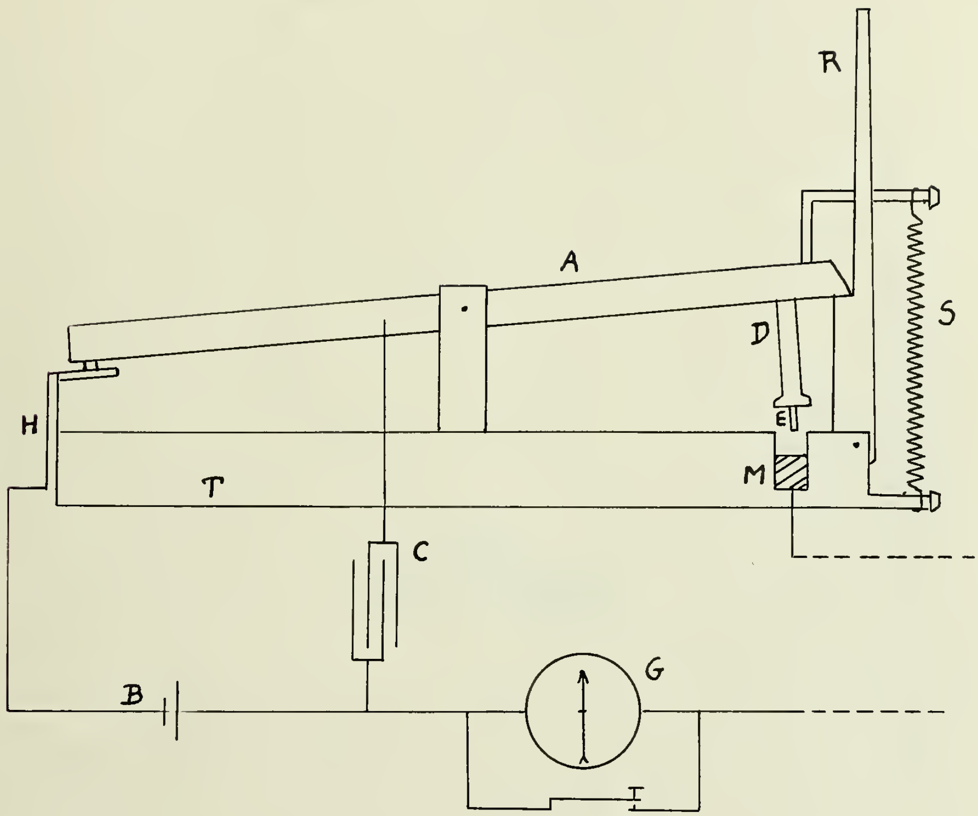


Fig. 4.

For opening and closing circuits a device was used which is common in the junior laboratory at the University of Illinois. It consists of a pendulum appratus as shown in Figure 2 in which a. is a wooden bar having as a cross-piece b. a brass cylinder filled with lead which swings at a given distance above the wooden arcs C and C', on which are placed keys like that shown in Figure 3. E is a screw for clamping the key on one of the arcs C, C'. Binding post a. connects with d., a strip of spring brass which makes contact with bdining posts b, or c, according as detent bar f. is up or released. Now by putting on each arc as many keys as are necessary for the purpose they can be so connected up that the pendulum striking the release bars will make or break or change the circuit as is desired. By placing one key, each on the opposite arcsC,C' they can be so set as to make the interval between their succesive release short at will even down to .0004 second. There is an objection, however, to the use of these keys, especially for closing the circuit, which Professor Zeleny points out. He indicates that the time required for discharge is about twice what it should be theoretically and this, he says, is due no doubt to the fact that the spring on striking the upper contact, instead of making a continuous connection vibrates against it for a short time. This difficulty he partially remedied by a mercury contact key like that shown in diagram in Figure 4, where T is a block

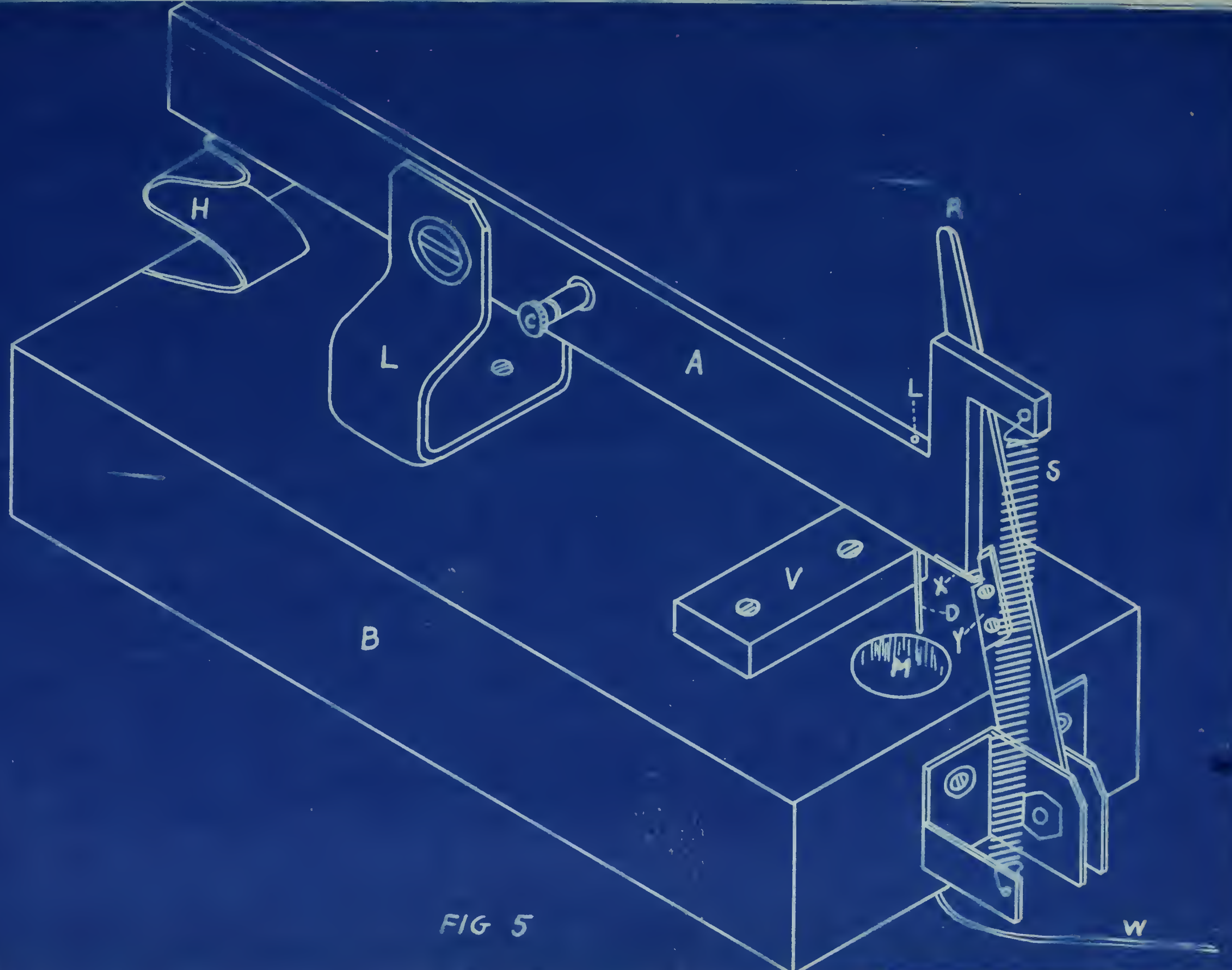


FIG 5

of insulating material, M a mercury cup, a. a bar of conducting material with a fulcrum at the center, one end resting on the spring H and the other fitting in a notch on the upright detent bar R. Spring S. is connected between A and T which, working in conjunction with H, causes the amalgamated point E. of Bar D. to plunge into the mercury cup when R is tripped by the pendulum. There is a disk just above the point E to prevent the spalshing of the mercury. The galvanometer G., battery B, and condenser C are connected up as shown in the figure. Thus the battery-condenser circuit is broken just an instant before the condenser-galvanometer circuit is closed. This allows no rebounding after contact is once made.

The key used in this thesis was varied slightly from the one just described. Figure 5, shows an isometric drawing of the key. B is a block of hard pine, dry and well seasoned. The mercury cup M was coated several times with shellac to insure good insulation. W. is a wire having an amalgamated point entering the cup M from the bottom and furnishing one of thegalvanometer circuit terminals. H. is a brass spring made in that shape so that it would be less stiff and yet give the aluminum bar A plenty of motion by following it farther. H. Carries a binding post E at its lower end which furnishes one terminal for the battery circuit, binding post c being the other terminal common to both the battery and galvanometer

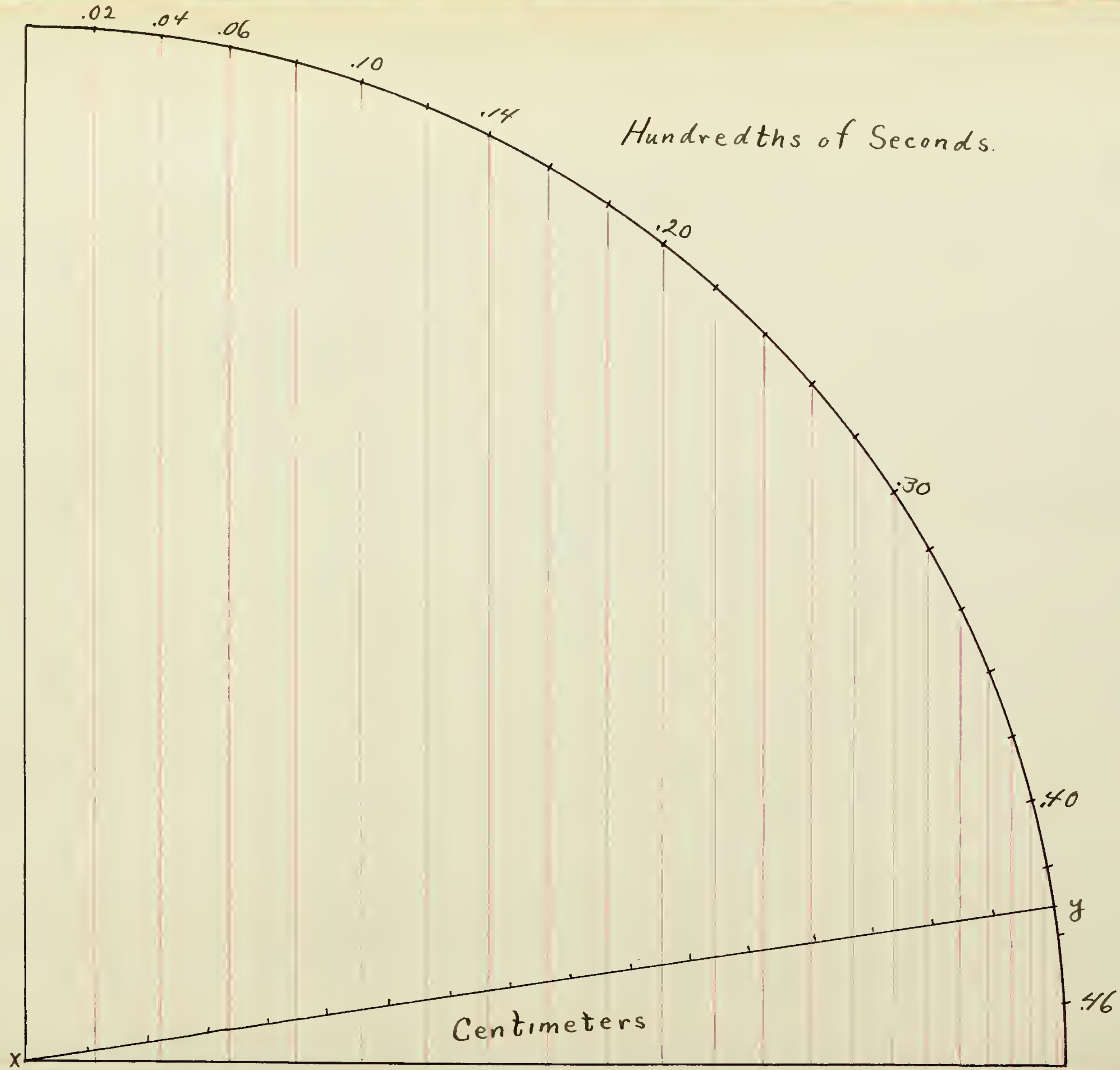

.02
.04
.06
.10
.14
Hundredths of Seconds.
.20
.30
.40
y
.46
x
Centimeters

Fig. 6.

circuits. At first,instead of steel strip X and steel block Y, the detent consisted of the bare aluminum bar A on a brass piece projecting from R. The corners, however, soon wore round and then it would not trip the same way each time it was released. V. is a block of soft wood which prevented X from striking on B, thereby bending or dislocating it. D. is an amalgamated copper wire put through a hole in A and headed at F. This it was found, needed no disk to prevent the splashing of the mercury. R. was made of aluminum in order that it would be light and not impede the pendulum unduly.

The scheme for calculating the small intervals of time is one that is in common use. The period of the pendulum was determined by counting and timing a large number of swings. The quarter period was then found and expressed in hundreths of seconds. A quadrant of a circle was then drawn (Figure 6), the radius of which was equal to the maximum displacement of the first swing of the pendulum, when relased from detent d. Then assuming that for the arc, through which the pendulum goes, its motion is simple harmonic, if we divide the quadrant in Figure 6 into as many equal parts as there are hundreths of a second in the quarter period of the pendulum and if lines be drawn from these points on the circumference of the quadrant of the circle perpendicular to the horizontal diameter, the intercepts will mark very closely the space moved over by

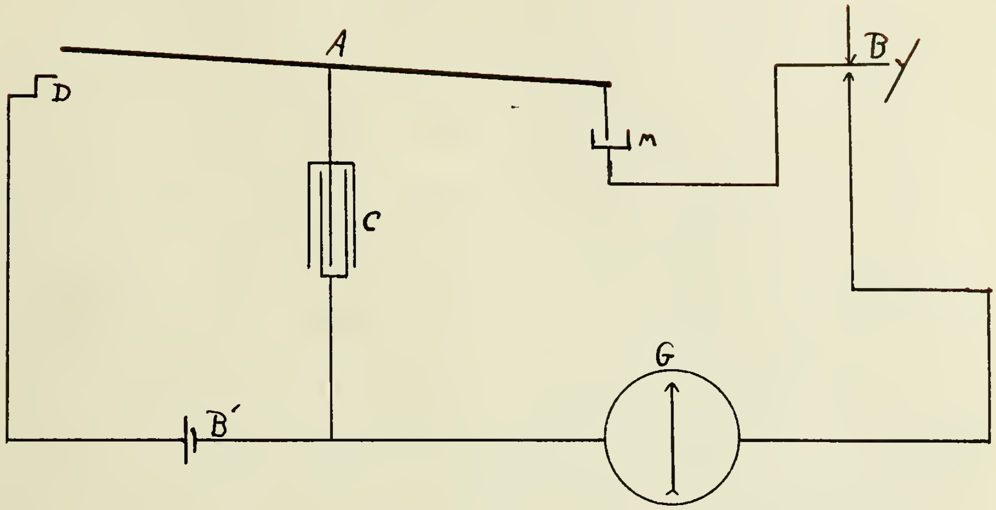


Fig. 7

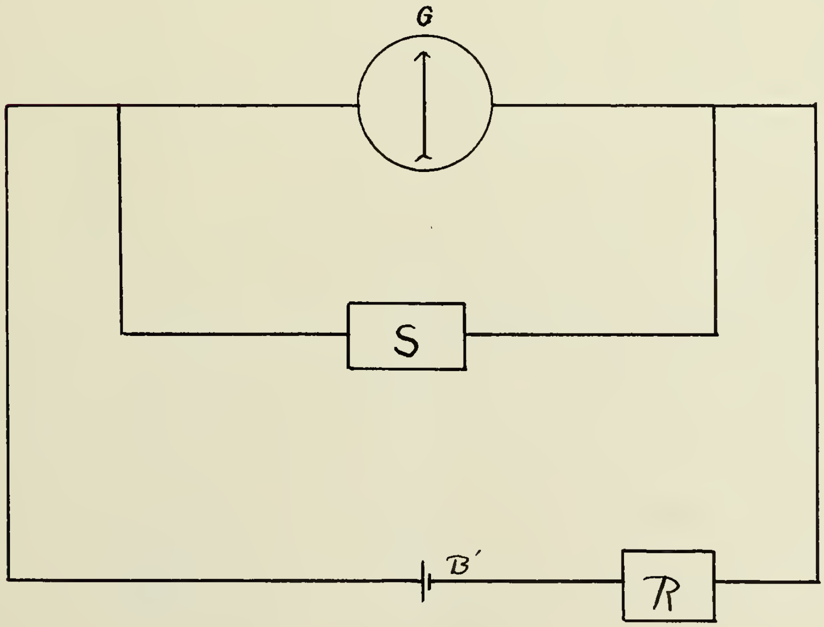


Fig. 8.

the cross bar of the pendulum in each corresponding hundreth of a second. Now if a line be drawn such as X, Y having approximately the same rise as the wooden arcs of the pendulum apparatus in the same distance from the center, the intercepts on X,Y made by these perpendicular lines, will be a still closer approximation. If this figure is drawn to full scale, the divisions on X,Y corresponding to a hundreth of a second can easily be estimated to millimeter so that a millimeter in the first division would men 1/13 of .01 second which is .00077 sec. Figure 6, is drawn to about half scale.

METHODS

The first of the various methods of measuring capacity investigated was that given by Carhart and Patterson* as the "First Method". It depends for the vlues used in calculation on a ballistic throw of the galvanometer due to the discharge of a condenser throught it and then a steady deflection with the same E.M.F. and a resistance in circuit. The set-up for the ballistic throw is shown in Figure 7. A and B are diagrammatic sketches of Figures 5 and 3 respectively. The key shown in Figure 3 may be used here as its function is only to break the ciruuit.

If Q is the quantity of electricity and A, $= \frac{H}{G}$, the galvanometer constant, T the half complete period of the coil, θ the first angular throw, and λ the logarithmic decrement, then

$$Q = \frac{AT}{\pi} \left(1 - \frac{1}{2} \lambda \right) \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\; (1)$$

If d is the value of the throw in millimeters and a is the distance of the scale from the galvanometer mirrow in millimeter we have

$$\theta = \frac{d}{2a}$$

Hence (1) becomes

$$Q = \frac{AT}{2\pi a} \left(1 \times \frac{1}{2} \lambda \right) d \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\; (2)$$

Now if a condenser is subject to an E.M.F. equal to E and it has a capacity C, then by the definition of capacity we have

$$Q = CE$$

Substituting in (2) and transposing we get

$$C = \frac{AT \left(1 \times \frac{1}{2} \lambda \right) d}{2 \pi a E} \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\; (3)$$

* Electrical Measurements - Carhort and Patterson, p.227

If now we use the same E.M.F. in a set up like that shown in Figure 8 and get a steady deflection d_1 where R is the effective resistance of the circuit, we have, by Oliver's law, provided d_1 is small,

$$\frac{E}{R} = \frac{Ad}{2a}$$

Then

$$\frac{A}{E} = \frac{2a}{d_1 R}$$

Substituting this value of $\frac{A}{E}$ in equation (3) we get

$$C = \frac{2a}{d_1 R} \; \frac{T\left(1 + \frac{1}{2}\lambda\right)d}{2\pi a}$$

or

$$C = \frac{T\left(1 + \frac{1}{2}\lambda\right)}{\pi R} \; \frac{d}{d_1} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

In getting d the keys A and B, (Figure 8) are set so that the interval allowed for discharge is that represented by the abscissae of the points m, n, o and p in Figure 1. corresponding to curves A,B,C and D respectively, that is, the "free charge" capacity is desired. Data were also taken, for comparison, for the time interval equal to the quarter period of the galvanometer, that is, for this galvanometer 1.46 seconds. The position of the keys for

Standard Mica Condenser------------.5 M.F.

	1/4T)d	d	d_1	R_1	S	R	1/4T)C	C
	50	50	65.5	100000	30	1764176	.4075	.4075
a	50	50	40.5	110000	20	2855512	.4072	.4072
	50	50	30.7	110000	15	3770534	.4068	.4068
	50	50	16.7	110000	8	6972960	.4044	.4044

Parafin Condenser-----------------.? M.F.

	58.5	57	65.5	100000	30	1764176	.4769	.4646
b	58.5	57	40.5	110000	20	2855512	.4764	.4642
	58.5	57	30.7	110000	15	3770534	.4760	.4638
	58.5	57	16.7	110000	8	6972960	.4731	.4610

Paper Condenser-------------------.5 M.F.

	77	61	65.5	100000	30	1764176	.6276	.4972
	77	61	40.5	110000	20	2855512	.6275	.4968
c	77	61	30.7	110000	15	3770534	.6271	.4963
	77	61	16.7	110000	8	6972960	.6228	.4938

Old Condenser---------------------.? M.F.

	74	57	65.5	100000	30	1764176	.6032	.4646
	74	57	40.5	110000	20	2855512	.6026	.4642
d	74	57	30.7	110000	15	3770534	.6021	.4638
	74	57	16.7	110000	8	6972960	.5985	.4610

Paper Condenser-------------------.5 M.F.

	236	190.5	69	200000	20	5191012	.6205	.5009
	236	190.5	21.3	200000	6	16835122	.6198	.5003
e	236	190.5	20.5	1000000	30	17634179	.6148	.4963

TABLE II

	1/4T)d	d	d_1	R_1	S	R	1/4T)C	C.
				Old Condenser-------------------.? M.F.				
f	231	177	69	200000	20	5191012	.6074	.4654
	231	177	21.3	200000	6	16835522	.6067	.4648
	231	177	20.5	1000000	30	17634179	.6018	.4611
				Standard Mica Condenser----------1.0 M.F.				
	50	125	65.5	100000	30	1764176	1.0188	1.0188
g	50	125	40.5	110000	20	2855512	1.0180	1.0180
	50	125	30.7	110000	15	3770534	1.017	1.017
	50	125	16.7	110000	8	6972960	1.0011	1.0011
				Parafin Condenser----------------.? M.F.				
	58.5	121	65.5	100000	30	1764176		.9862
h	58.5	121	40.5	110000	20	2855512		.9854
	58.5	121	30.7	110000	15	3770534		.9845
	58.5	121	16.7	110000	8	6972960		.9786

Period-------------5.86 Seconds

Galvanometer Resistance----- 499 ohms

Logarithmic Decrement-------.0196

E.M.F.-----------Three dry cells.

TABLE II (continued)

the time equal to zero was found by trial. Key B was then moved forward from this zero position the required distance as shown in Table I and Figure 1. In the case of the condensers having a high absorbing power the keys were left "set" long enough (ie the circuit was left closed long enough) before releasing, to insure the discharge being independent of the time of charging or at least a very close approximation to it.

In finding R the effective resistance of the circuit in the case of Figure 8, the multiplying power of the shunt must be taken into consideration. The effective resiatnce of the circuit then is

$$R = \left[R_1 + Rb + \frac{Sg}{S+g}\right] \frac{g+S}{S}$$

This then substituted in equation (4) gives

$$C = \frac{T\left(1 + \frac{1}{2}\lambda\right)}{\left[R_1 + \frac{Sg}{S+g}\right]\frac{g+S}{S}} \quad \frac{d}{d_1} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

and this is the value of the capacity in forads. Data for this method appears in Table II.

In Table II sets, a,,b,c and d are all alike except values of d and the corresponding values of C. In each set there are four different values of d_1 each giving a different value of C and, since it was pointed out to

Nalder Bros. Galvanometer, Pl. 2002

	$\frac{1}{4}T)d$	d	d_1	R_1	S	R	$\frac{1}{4}T)C$	C
	Standard Mica Condenser--------.7 M.F.							
	218.3	216	76.5	300000	20	11064885	.75222	.74430
a	218.3	216	15.5	300000	4	54124330	.71755	.71000
	218.3	216	15.7	1100000	15	53725709	.71362	.70616
	Parafin Condenser------------.? M.F.							
	145	140	76.5	300000	20	11064885	.47238	.45609
b	145	140	15.5	300000	4	54124330	.47663	.46019
	145	140	15.7	1100000	15	53725709	.47405	.45770
	Paper Condenser---------------.5 M.F.							
	197	151	76.5	300000	20	11064885	.64179	.49193
c	197	151	15.5	300000	4	54124330	.64756	.49635
	197	151	15.7	1100000	15	53725709	.64405	.49366
	Old Condenser----------------.? M.F.							
	185	137.5	76.5	300000	20	11064885	.60270	.44795
d	185	137.5	15.5	300000	4	54124330	.60811	.45197
	185	137.5	15.7	1100000	15	53725709	.60482	.44952

Galvanometer Resistance--------718

Logarithmic Decrement----------.1073

Period of Coil----------------164/9 Seconds.

TABLE III

make d_1 small, it is assumed that the value of C corresponding to the smallest value of d_1 is the nearer correct. However, we find even in this value of C an error of 1% in the case of the standard mica condenser. Now d in set (a) is 50 and an error of one or two millimeters in reading it would make an error in the result of from 2% to 4%. If d is over 100 then an error of one millimeter would introduce an error of something less than 1%. Set g contains the same data as set a. except that d is 125 instead of 50 and now the value of C corresponding to the lowest value of d_1 is in error by .1 of 1%. This same general rule holds all through Tables II and III. In sets e and f (Table II) the values of d_1 are nearly the same in the second and third determinations, yet, it can be seen, that they were gotten by different means and that they give different values for C. The second was gotten by a low value of S and a low value of R, while the third was gotten by a high value of S and a high value of R_1. This is practically the only difference between the two determinations, yet we find the results different. Then one of them is nearer right than the other. If we compare the results in the third determinations with results obtained under a different and, also a very reliable method, we find that they agree more closely than the results of the second determination where R was gotten by low values of S and R_1. This

then, would indicate that very low values of S and R_1 and to be avoided. This should be expected, however, since a slight mistake in reading d_1 would be magnified more when the multiplying power of the shunt is high than when low values of S are used. The lower the value of S the greater the multiplying power of the shunt. Therefore, for accuracy it is best to keep S as large as possible and cut down the current through the galvanometer, by a greater series resistance.

The column labeled ($\frac{1}{4}$ T)d contains the throws of the galvanometer when the condenser is left in circuit with it during its entire quarter period, and the column (($\frac{1}{4}$ T)C) contains the corresponding values of the capacity. In the case of the standard mica condenser there is no difference between the two which in fact is what we should expect from an examination of curve A, in Figure 1, and from the fact that the columns labeled (($\frac{1}{4}$ T)d) and d are alike. In set (b) we find a slight difference between these two columns and from curve B)Figure 1) we should expect a small difference. This is manifest in the two columns containing the values of the capacities. A difference of about 2.5% exists, then, in the case of the parafin condenser, between the "free charge" capacity and the "ordinary" capacity. Correspondingly in the case of the paper condenser and the old condenser we find differences of from 28% to 30%.

In Table III, which was obtained in a like manner to Table II except that a Nalder Brothers' galvanometer was used instead of a type H, it will be seen that a slight difference between "ordinary" and "free charge" capacity exists in the case of the standard mica condenser. This shows that if Curve A were produced to the point corresponding to $4\frac{1}{9}$ seconds, the quarter period of the Nolder Brothers' galvanometer, it would show some rise. There is a difference, in this table, between the two capacities of a little over 1%. In the case of the parafin this difference is about 3.5% while in the case of the paper and old condensers the difference is from 33% to 35% and the difference between the corresponding values of Table II, sets e and f, and the values of $(\frac{1}{4}T)C$ in Table III sets c and d amounts to from about 1% to 5%. That is, the value of the ordinary capacity may vary by as much as 5% according to the period of the galvanometer. The "free charge" capacity in the same instances vary from 1/2% in the paper to 2 1/4% in the old condenser, taking the third determination as the nearest right.

The Nalder Brothers galvanometer was found to be rather inaccurate for several reasons. Its period was so long and the movement of the coil so slow that its resistance could not be accurately measured and g, the resistance of the galvanometer is a factor in the equation. Then, too, its decrement was large and varied greatly from the wide

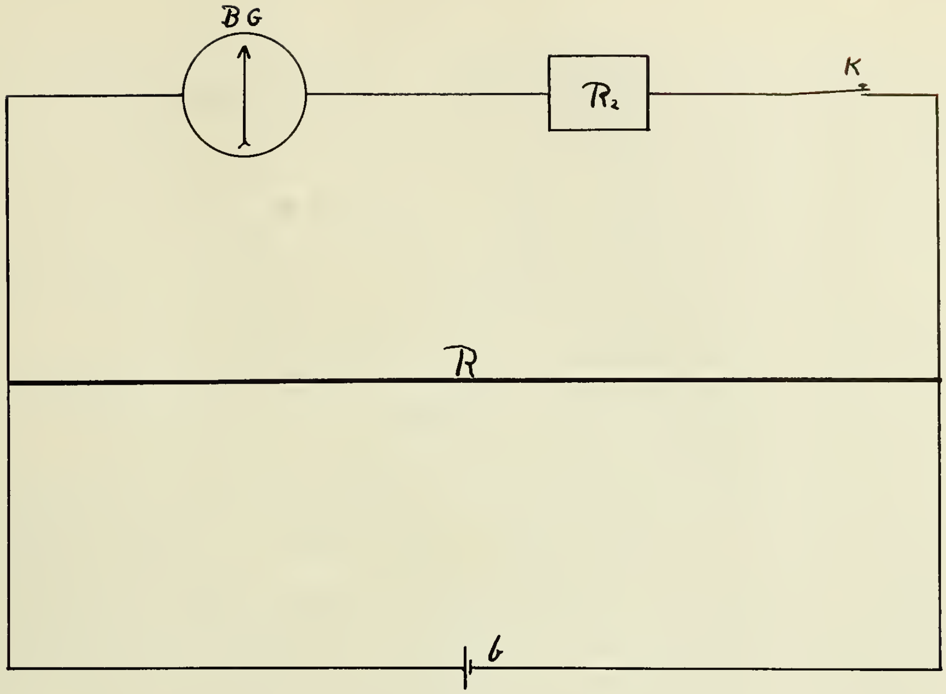


Fig. 9.

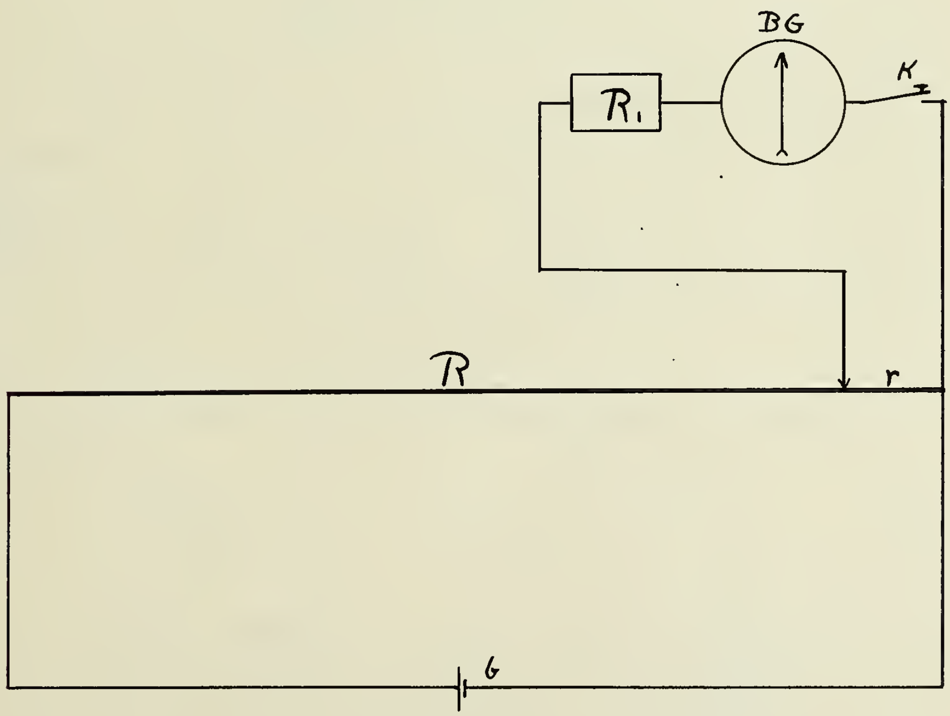


Fig. 10.

to the small amplitudes. These factors were found sufficient to account for the errors in the results obtained with it. The curves of Figure 1 were referred to for the value of the time interval for discharge in getting d.

The other method to be investigated was the one due to Herderson.* It depends on a ballistic throw of the galvanometer on the charge of a condenser and then a steady deflection with a known fractional part of the same E.M.F. For the ballistic throw connections are made as shown in Fig. 9, R is a very high resistance to the ends of which a battery is connected and from the ends of which a circuit is led off containing a ballistic galvanometer B.G.,a condenser C, and a high resistance key K. On closing K, electricity will flow into the condenser casuing the galvanometer to deflect in proportion to the quantity passed through it to the condenser. A circuit containing a tapping key is connected across the terminals of the condenser for the purpose of discharging it.

By the definition of capacity we have

$$C = \frac{Q}{V}$$

where C is the capacity, Q the quantity of electricity, and V the drop of potential across the terminals of R which is so high a resistance as to represent the total fall of potential of the battery or the total E.M.F. of the battery.

*Practical Electricity and Magnetism - Henderson, p.232

The value of Q in terms of the galvanometer constant $\frac{H}{G}$, the angular deflection , and the period of the galvanometer T, can be written down from the well known equasion.

$$Q = \frac{10\ H\ T}{2\ \pi\ G}\ 2 \sin \frac{\alpha}{2}\ \left(1 - \frac{\lambda}{2} \right)^{*} \ldots\ldots \quad (10)$$

Now, to get the value of V, the terminals of the galvanometer having a high resistance in series with it, are connected over r. a known fractional part of R as in Figure 10. This gives a steady deflection (S) of the galvanometer, which the author says should not be very different from . Adjustment of S can be made by altering R_1 or r. If c is the current through the galvanometer and e the fall of potential over r. we have

$$C = \frac{e}{Rg + R_1} = 10\ \frac{H}{G} \tan.d - \ldots\ldots \quad (10)$$

$$\text{Then } e = 10\ \frac{H}{G}\ (R_g + R_1) \tan d \ldots\ldots \quad (12)$$

but V:e::R:r

$$\text{or } V = \frac{eR}{r} \ldots\ldots \quad (13)$$

$$V = \frac{10\ (Rg - R_1)\ H\ R \tan d}{R\ \ G} \ldots\ldots \quad (14)$$

$$\text{Then } C = \frac{Q}{V}$$

* Practical Physics - Stewart and Gee, Part II,p.407

Substituting (10) and (14) in this we get

$$\frac{C = \dfrac{10\ HT}{2\pi G}\ 2\sin\dfrac{\alpha}{2}\ \left(1 + \dfrac{\lambda}{2}\right)}{\dfrac{10\ (R_g + R_1)\ H\ R\ \tan S}{r\ G}}$$

$$\text{or}\quad C = \frac{Tr\ \sin\dfrac{\alpha}{2}\ \left(1 + \dfrac{\lambda}{2}\right)}{\pi R\ (R_g + R_1)\ \tan S}$$

This equation gives the capacity in forads.

When the deflections were large it was found necessary to correct for the tangent and sine of the angle of deflection. In this work a type H galvanometer having a telescope and circular scale, was used and for the larger throws half the scale reading was taken as the measure of the angle in radians and this was reduced to degrees and then the tangent or sine obtained from a table of natural functions. In the case of a straight scale, one half the deflection in millimeters divided by the scale distance from the galvanometer mirro would be the value of the tangent of the angle of deflection. In the place of the authors high resistance key, a tapping key and a resistance box R_2 in series with the condenser and galvanometer, was used. The values being about as follows $R = 12100$, $R_1 = 10000$ and $R_2 = 5000$.

Now from Curve A, Figure 1, assuming that a curve plotted on charge is the same as one plotted on discharge, an assumption which will have to be modified later, we would expect that it would make no difference in the deflection, in the case of Figure 9 when using the standard mica condenser, whether the key K, was tapped or held down during the whole quarter period of the galvanometer. It was found, however, that it did make a difference. It was also found that, when R_2 was zero, it did not make a difference. Then the natural conclusion would be that R_2 was affecting the quantity of electricity which went through the galvanometer. Again, as we should expect from Curves C and D, (Figure 1) there was a difference in the throw according to the length of time the circuit was closed up to the value in time of the quarter period of the galvanometer, when using the paper or old condenser. This, however, as is readily understood, was due to the absorption in the dielectric. This latter fact was sufficient to show the necessity of breaking the circuit as soon as the amount corresponding to the free charge capacity has flowed in. But lest the curves plotted with time intervals of charging as abscissae and galvanometer throws as ordinates, should be different from those of Figure 1 in which time intervals of discharging were plotted against galvanometer throws, curves were plotted from data taken for different intervals of charging. The same scheme for

making and breaking the circuit and timing the intervals was used as in the first method. This data appears in Table IV and the curves in Figure 11. It will be seen that as the time interval is longer the curves B and C representing the paper and old condensers respectively, keep rising up to the quarter period of the coil just as in Figure 1, also that Curve A goes horizontal after a very short interval as in Figure 1 and again corresponding to points m, o and p, in Figure 1 there are points j, r, and l on the corresponding curves of Figure 11, the abscissae of which represent the time required for enough electricity to get into the condenser to raise it to the first maximum difference of potential. The phenomea of absorption is responsible for the subsequent rise of the curve. Then the points j, r, and l correspond to the free charge capacity which includes the free quantity plus the amount absorbed by the dielectric while the potential difference rises first to a maximum. Thus in measuring the capacity of a condenser by a ballistic throw of the galvanometer on charge, the keys must be set so as to get this time of charge.

The data for this method is shown in Table V.

Only a part of the data taken is here tabulated but enough is here to show that only an accidental set of conditions might givo correct results. Table V shows the results for only the standard mica condenser, but even these

are not consistent and not correct. High and low values of R and R_1 were used, and in connection with each, different values of r were used, giving different values of tan d. R_2 must be zero as we shall see later and hence is a certain definite quantity for a given capacity and must not be varied.

A given color of curve in Figures 1 and 11 represents the same condenser. The black line represents the standard mica condenser, the red line the paper condenser, the blue line the old condenser, and the green line the parafin condenser. The curves on charge seem to have a somewhat sharper turning point than the curves on discharge. The points marking the complete passage of the free charge are less mistakable. The black line on discharge is below the red line in the vicinity of the knee of the curve, while on charge the black line is above the red line in this region. This means, if we assume the mica condenser to be the more nearly perfect in efficiency that we get a greater quantity from the paper condenser than we put into it. But this is contrary to the principal of the conservation of energy and so we must assume that the paper condenser is the more nearly perfect one. Then the curves indicate by their relative positions in the two figures a loss of energy, at least in the mica condenser. That is, we have a hysteresis in the dielectric. Messurs. Porter and Morris* have obtained experimental evidence that seems to deny dielec-

*Proc. Roy. Soc. Vol. LVII p.469.

tric hysteresis. They admit a viscosity of the dielectric due to absorption, but their experiment shows no loss of energy. Time is required for the dielectric to assume its normal condition yet within this interval of time the efficiency is 100%. This can hardly explain the phenomena under consideration since the standard mica shows more viscosity than the paper condenser and if the viscosity is due to absorption in the dielectric, we have evidence from the curves that there is greater absorption in the paper than in the standard mica condenser. The problem still remains and this thesis only ventures to suggest it.

It is interesting to note in Figure 18 that the curves all cross one another at the same point, and while Curve A belongs one centimeter to the left, yet since it is horizontal in this vicinity, it would still cross the others at this point when corrected. It is also ture that the curves to the right of this point have the same relative positions with reference to one another that they do to the right of a similar point in Figure 1, ie., blue being uppermost then red, green and black in succession below.

Attention has been called previously in connection with the method due to Henderson, to the fact that the galvanometer throw is a function of the time the condenser is allowed to remain in the circuit when there was a high resistance in series with it. This fact suggested an investigation of the effect of a high resistance in the circuit.

A			B			C		
Arc scale reading	Calculated time in seconds	Galv. deflection in m.m.	Arc scale reading	Calculated time in seconds	Galv. deflection in m.m.	Arc scale reading	Calculated time in seconds	Galv, deflection in m.m.
396	0	0	396	0	0	396	0	0
397	.00077	90	397	.00077	76	397	.00077	75
398	.00154	126	398	.00154	117	298	.00154	98
399	.00231	126	400	.00308	118	400	.00308	102
402	.00462	126	405	.00693	120	405	.00693	103
410	.01077	126	420	.01848	122	420	.01848	115
450	.045	126	450	.045	124	450	.045	123
$\frac{1}{4}$ T	1.465	126	500	.0875	128	500	.0875	129
			575	.1564	131	575	.1564	134
			675	.2633	135	675	.2633	138
			$\frac{1}{4}$ T	1.465	150	$\frac{1}{4}$ T	1.465	146

Leeds and Northrup Galvanometer pl.? 2717 B

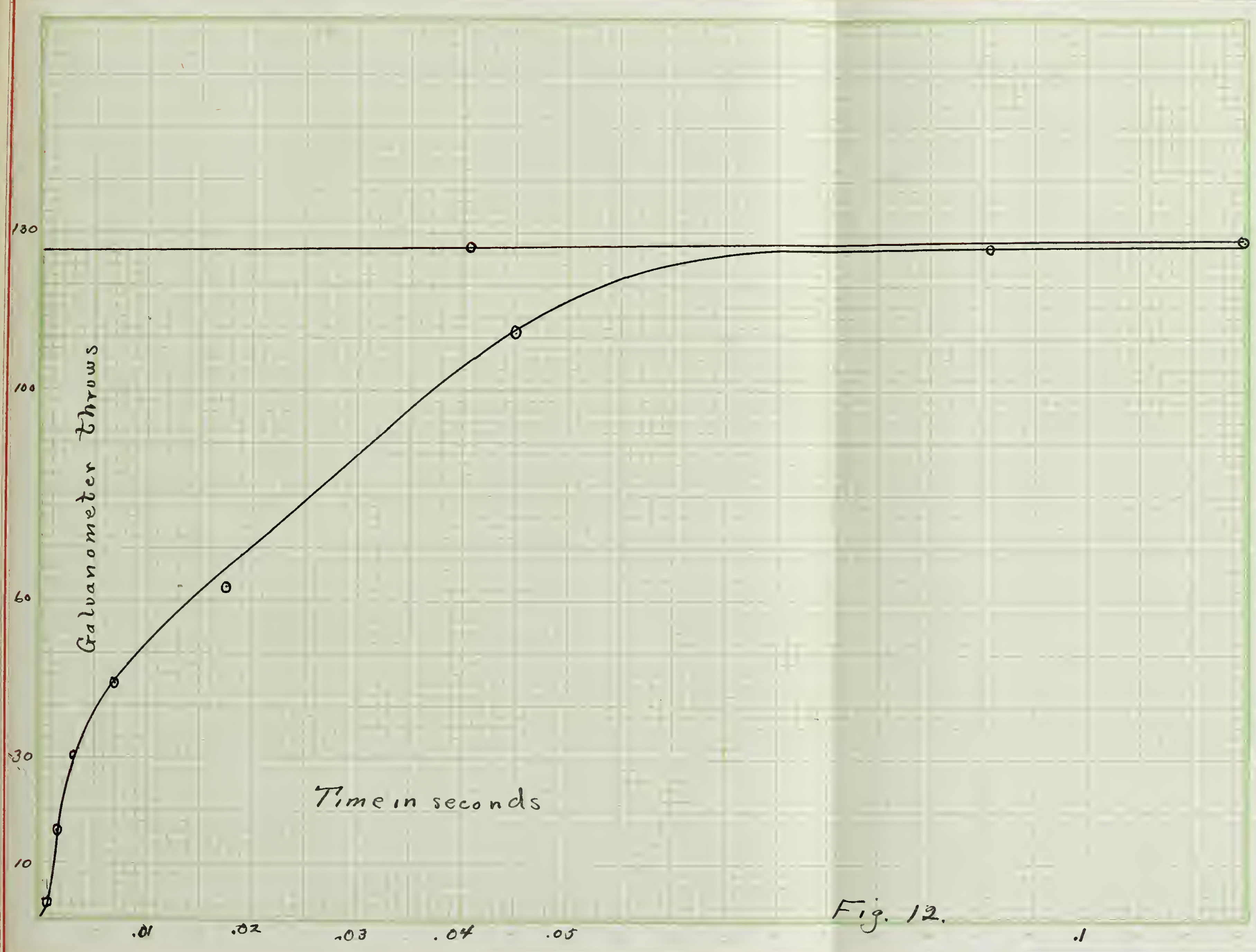
Rg = 490

= .0196

T = 5.86 sec

E.M.F. one dry cell

TABLE IV



130
100
60
30
10
Galvanometer Throws
Time in seconds
.01
.02
.03
.04
.05
.1

Fig. 12.

Henderson's Method

d	$d(1-2\lambda)^{1}$		$\sin\frac{\alpha}{2}$	R	R.	$d(\delta)$	$\tan\delta$	r.	C.
126	127.23	$7^{o}5.86$	.0622	22100	60 000	9.8	.009589	60	.5162
"	"	"	"	"	"	32.9	.03319	200	.51257
"	"	"	"	"	"	66.0	.06468	400	.51300
"	"	"	"	"	"	114.3	.11184	700	.51635
140	141.5	$7^{o}57.59$	.0699	11000	10 000	26.	.025713	10	.45722
"	"	"	"	"	"	52.2	.05158	20	.41562
"	"	"	"	"	3 000	77.4	.07648	10	.420556
"	"	"	"	"	3 000	149.4	.148755	20	.43574
"	"	"	"	"		30.8	.03043	4	.42272
12.8					10 000	102.2	.10084	50	.48669

Standard mica condenser

Leeds and Northrop type H. galvanometer

T = 5.86 = .0196 Rg = 499

E.M.F. = one dry cell

TABLE V

Table VI shows the result, when the circuit was opened and closed with a tapping key, of increasing the resistance. The column marked d_1 contains a record of the galvanometer throws for a quick tap of the key while d_2 is the result when the circuit is kept closed during the first quarter period of the galvanometer. The condenser used was a standard mica of 1/2 microforad, the E.M.F., was furnished by two dry cells, R was 12100 ohms, and Rg was 500 ohms.

Table VI then suggested setting up the pendulum apparatus as in the first method and setting the keys at such a distance apart on the arcs that the circuit was closed for a little longer than the time necessary for the full measure of free charge to pass, and the values for this interval were tabulated as d_1 (Table VII) while d_2, Table VII is as d_2 in Table VI. R_2 was increased to 160 000 before d_2 was affected, d_2 however, at R_2 = 2 megohms, was cut down to 93 from the value 128 when R_2 = 0. d_1 affected for R_2 = 1000 and was cut down from 128, when R_2 = 0, to 8 when R_2 = 2 000 000 ohms.

The results as shown by these two tables are after all just what might have been expected. It is quantity that affects the galvanometer and a very short yet finite length of time is required for its passage. But its rate of passage depends on the resistance of the circuit. Now, if the resistance is made so high that the quantity cannot all get through in the time allotted it,

R	d_1	d_2
0	125	125
3100	123	125
62000	114	125
122000	9 0	125

TABLE VI

R	d_1	d_2
0	128	128
10000	127	128
20000	127	128
40000	120	128
100000	86	128
120000	80	128
160000	66	127
200000	55	127
250000	48	126
300000	40	125
360000	35	124
460000	30	122
560000	23	121
760000	18	118
1000000	14	112
2000000	8	93

TABLE VII.

Leeds and Northrup Type H. Galvanometer Pl. 2717 B.

$T = 5.86$ $\lambda = .0196$ $R_g = 499$

E.M.F. = One dry cell.

Arc	Time-Sec.	Galv. Defl.
398	0	0
399	.00077	3
400	.00154	16
402	.00308	32
407	.00693	45
422	.01848	62
452	.045	111
502	.0875	127
577	.1564	127
677	.2633	128
1/4 T	1.465	128

TABLE VIII.

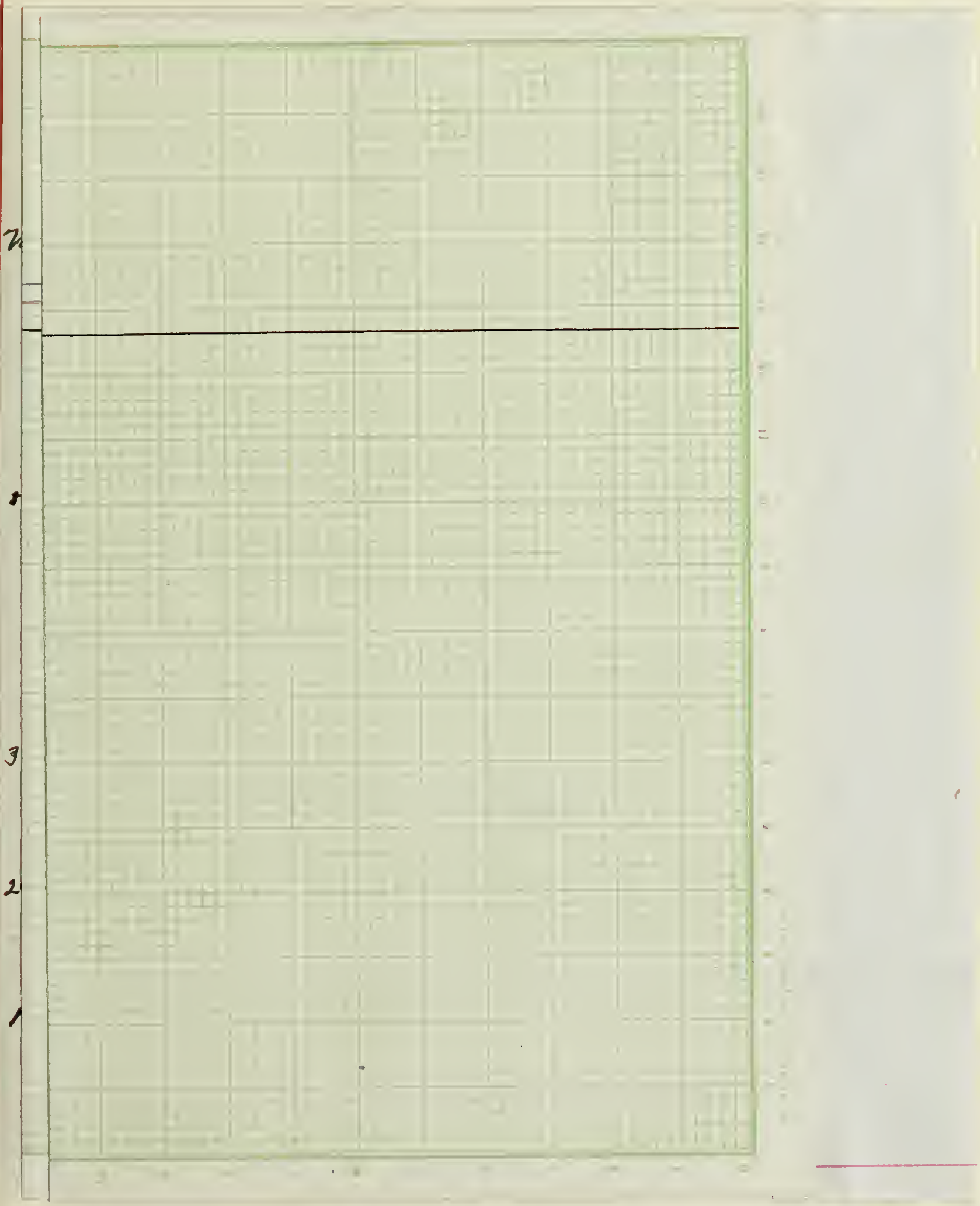
Leeds and Northrup Type H. Galvanometer Pl. 2717 B.

Period of Coil = 5.86 Seconds

Logarithmic Decrement = .0196

Galvanometer Resistance = 499 ohms

E.M.F. = One dry cell.



3
2
1

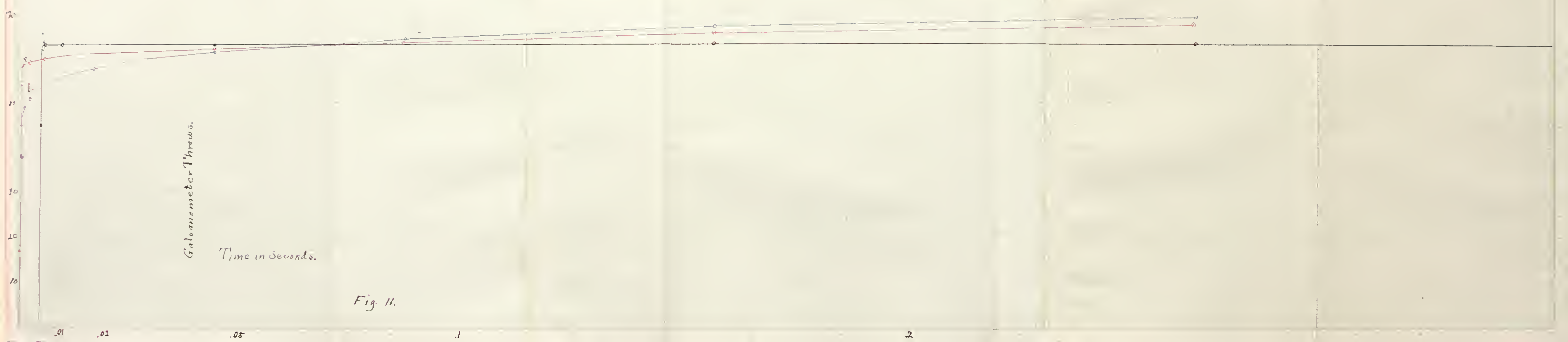


Fig. 11.

the actual quantity passing through the galvanometer would be cut down and correspondingly the throw of the coil would be affected. The movement of the coil is not the same for a given quantity going through it in the form of a continued current as when it is all sent through before the coil has moved through an appreciable angle. So when the resistance is high, causing the quantity to go through slowly, the formula for the ballistic galvanometer will not necessarily hold. In the measurement of capacity then by ballistic methods, we should expect a high resistance to have the same effect as a high absorbing power in the dielectric, since it affects the galvanometer in a similar way.

With this in mind data (Table VIII) was taken and a curve (Figure 12) was plotted for a standard mica condenser having in series with it a resistance of 20 000 ohms. This curve was plotted on charge as in Figure 11 in which curve A represents the same condenser, having no resistance in series with it except that of the galvanometer, as the one used for this curve. This curve after reaching a certain heighth is practically horizontal. The earlier part of the curve indicates that the current is one of gradually diminishing strength from a maximum at the beginning. If the current were steady from the close of the circuit till all the quantity had passed, the throw of the coil would vary as the time of charge and the curve

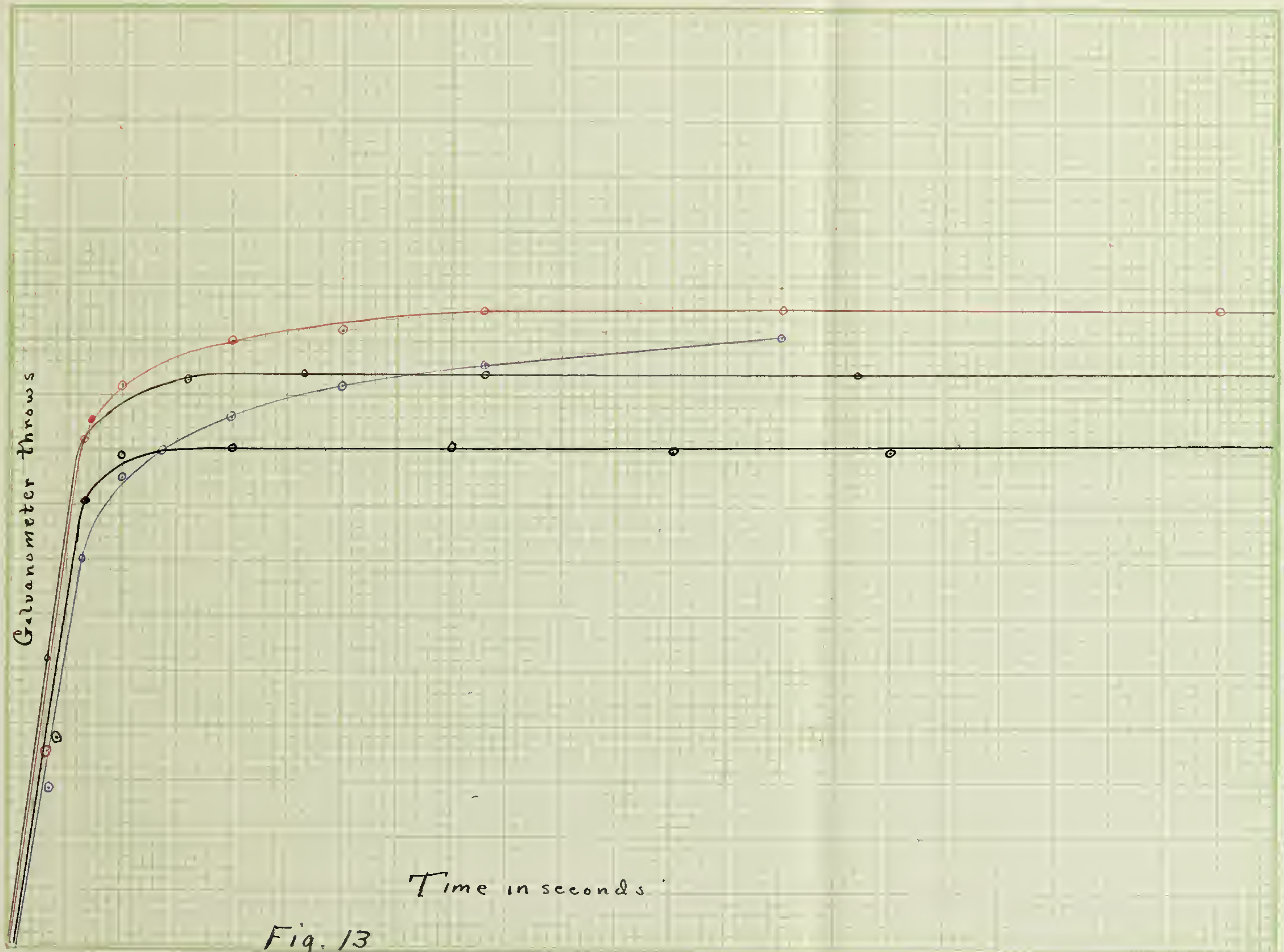


Fig. 13

would be a straight line. But this is not the case. It is evident, then, that below a certain point, which we might call the critical point and which marks just the amount of time required for the whole free charge quantity to pass, the series resistance produces the same effect as an absorbing power in the dielectric and hence for accurate measurements this resistance should be as low as possible and, for the same reason, a low resistance galvanometer is preferable to one of high resistance. The effect of a galvanometer of higher resistance is shown in Figure 13. Note that the first points above the origin on each curve, are off the curve to the right. No reason could begiven for this until it was remembered that the galvanometer used, had a resistance of 1220 ohms. These points are evidently below the critical point while the next ones are above it. It is probable then that in plotting the variation of ballistic galvanometer throw with time of charge or discharge, unless the resistance is so low that the critical point on the curve comes below the smallest measureable interval of charge or discharge, we do not have a true representation of the time required for a condenser to discharge its free charge quantity. Theoretically then, any resistance whatever in the galvanometer or its circuit would retard the passage of the free charge quantity. This, of course, cannot be shown experimentally unless we could measure infinitely small intervals of time. Professor Zeleny suggests

that even after he had constructed his mercury contact key, that the time necessary for the discharge of the whole measurable part of the free charge was still somewhat greater than that demanded by theory.* It might be suggested here that this extra time demanded, was due to a high resistance galvanometer or, at least, high enough to affect the discharge of the free charge.

SUMMARY

The ordinary capacity of a condenser of a high absorbing dielectric, measured by a ballistic galvanometer is greater than the free charge capacity and does not represent the ture capacity. The free charge capacity is obtained by breaking the condenser galvanometer circuit as soon as the free charge is liberated. When this correction is applied to the first method, it can be used to measure the capacities of condensers having a high dielectric absoprtion, with a reasonable degree of accuracy.

The method given by Henderson, so far as this investigation is concerned, was found unsatisfactory, but its consideration has brought up a number of interesting questions. This method demands the same correction as the first method

*Phys. Rev. Vol. XXVII p.66

for capacities of high absorbing dielectric. We found that a comparison of the curves plotted on charge and on discharge indicate a loss of energy in the dielectric, that is, a dielectric hysteresis and this cannot be explained by Porter and Morris' article in which they deny such a thing as dielectric hysteresis but admit a viscosity of the dielectric due to absorption, since the condenser having the least absorption shows the greater apparent hysteresis.

We have seen by a comparison of the curve obtained from the old condenser with that of a curve obtained by Professor Zeleny from an old condenser, experimental evidence of the truth of the statement by Professor Nipher, that the specific inductive capacity of a dielectric is a function of its entire previous history.

This thesis was worked out with the apparatus in use in the junior laboratory at the University of Illinois. The resistances were standard made by Hartman and Braun and the standard condenser was one of Leeds and Northrups' standard mica condensers.